Self-tuning of cosmological constant and exit from inflation *

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I review the recent 5D self-tuning solutions of the cosmological constant problem, and try to unify two cosmological constant problems within the framework of the self-tuning solutions. One problem, the large cosmological constant needed for inflation, is interpreted by starting with the parameters allowing only the dS vacuum, and the vanishing cosmological constant at a true vacuum is realized by changing parameters by exit from inflation at the brane such that the self-tuning solution is allowed.

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1. Introduction

In particle physics, there are three vacuum energies to deal with: (i) the GUT scale vacuum energy for inflation, (ii) the vanishing cosmological constant at the minimum which is theoretically favored, and (iii) the observed tiny vacuum energy $(0.003 \text{ eV})^4$ at $z = 10 - 100$. The tiny vacuum energy discovered from Type 1A supernova data and confirmed by the WMAP data makes the cosmological constant problem more difficult. Here, we do not discuss this tiny vacuum energy. Simply, we assume that somehow this tiny vacuum energy is resolved by a (pseudo)scalar field [1] whose misalignment shifts the vacuum energy a little bit from a true minimum of the vanishing vacuum energy with $\omega = p/\rho \leq 0.8$. The cosmological constant($\Lambda = 8\pi G_N V_0$) is a term in Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 8\pi G_N V_0 g_{\mu\nu} = 8\pi G_N T_{\mu\nu}. \tag{1}$$

The LHS of (1) without Λ vanishes when the spacetime is flat, $g_{\mu\nu} = \eta_{\mu\nu}$. Thus, if matter is present, the RHS of (1) is nonvanishing and hence spacetime is not flat, i.e. $g_{\mu\nu} \neq \eta_{\mu\nu}$. Therefore, under a reasonable assumptions, Eq. (1) must lead to an evolving universe. In 1910's, the universe looked as if it were a static one. So, Einstein introduced a compensating term, $\Lambda g_{\mu\nu}$, on the LHS of (1) to make the solution static. This is the birth of the cosmological constant. In this talk, I will neglect matter(i.e. quantum excitations), and hence we are looking for the vacuum

solution. Without matter but with a nonvanishing cosmological constant Λ, it is not possible to have a static solution.

Probably an easy way to see the cosmological constant(c.c.) problem is to start with an action where it is easy to find out possible symmetries. The LHS of the Einstein equation with the c.c. is obtained from the following action

$$S = \int d^4x \sqrt{-g} \left(\frac{M^2}{2} R - V_0 \right) \tag{2}$$

where g is the determinant of the metric tensor, R is the Ricci scalar, M^2 is the reduced Planck mass $M^2 = 1/8\pi G_N$, and V_0 is a constant which is related to the cosmological constant by $V_0 = M^2 \Lambda$.

The difficulty with the c.c. problem is that there is no symmetry working for it to vanish. An obvious symmetry is the scale invariance, but it is badly broken. At the electroweak scale, we need a mass parameter of order 100 GeV, so the c.c. is expected to be 10^{56} times larger than the current value. This cosmological constant problem surfaced as a very severe one when the spontaneous symmetry breaking of the electroweak theory was extensively discussed [2].

The Einstein equation can be solved with an appropriate ansatz for the metric. For example, a flat space ansatz is $g_{\mu\nu} = \eta_{\mu\nu}$, and a de Sitter(dS) space ansatz is

$$ds^2 = -dt^2 + e^{2\sqrt{\Lambda}t} d\mathbf{x}^2. \tag{3}$$

In 4D, the flat space is possible with the vanishing vacuum energy. Thus, to have a flat space one has to fine-tune the parameters in the action such that the vacuum energy is exactly zero. To have a flat 4D space with a finite range of parameters in the action, one must go beyond 4D.

Our 4D may come from higher dimensions. In this regard, the Randall-Sundrum(RS) type models [3] are very interesting. These are most easily studied in 5D. It will be sufficient if we, the 4D observers, see the vanishing effective 4D c.c. even though we start with nonzero 5D c.c. Indeed, the RS-II [3] model starts with a negative 5D c.c. Λ_b(bulk c.c.), AdS_5, and a 3-brane(s) with brane tension Λ_1, but allow flat 4D, as shown by the line element

$$ds^2 = \beta^2(y)(-dt^2 + d\mathbf{x}^2) + dy^2 \tag{4}$$

where $\beta(y)$ is the warp factor. If there exists a reasonable solution with the line element (4), then it describes a flat space. It is usually assumed that matter is present at the brane B1 located at $y = 0$. Indeed, a flat 4D is possible with one fine-tuning, $k_b = k_1$, with the exponentially suppressed warp factor for large y; thus the 5th direction is not noticeable to the 4D observer of B1 and there can results a consistent 4D. An important thing to note is that a flat 4D is obtained, starting with nonzero c.c. But still a fine-tuning between parameters is needed.

2. Self-tuning solutions

Strong self-tuning solution: Without fine-tuning there exists a flat-space solution which is not continuously connected to dS or AdS solutions. It was tried by Stanford groups [4], but soon it has been shown that it has either the singularity problem or reintroduces a fine-tuning [5]. The solution they obtain has a singularity at say $y = y_c$. This is because to cure it, or to satisfy the sum rule, one introduces another brane at $y = y_c$. As soon as one introduces a brane, there is one more parameter introduced there, namely the brane tension. A flat space is possible only for a specific value of the new brane tension, needing a fine-tuning. So there does not exists an example for the strong self-tuning solution.

Weak self-tuning solution: This solution does not question whether the flat solution is connected to dS or AdS solutions. Of course, the flat self-tuning solution does not require a fine-tuning(s). It was proposed in early 80's with antisymmetric tensor field strength $H_{\mu\nu\rho\sigma}$. But because that was done in 4D, the idea was not realistic. However, in 5D the weak self-tuning solutions can be made realistic in RS-II models.

In RS-II type models, indeed there exist weak self-tuning solutions [6,7]. One solution employs the antisymmetric tensor field A_{MNP} in 5D AdS_5 and one brane located at $y = 0$. The bulk c.c. is Λ_b and the brane tension is Λ_1. If one introduces the standard term H^2, there does not exist a self-tuning solution. In fact there exists the no-go theorem with a standard kinetic energy term [8]. The authors of Ref. [5] used $1/H^2$ term where $H^2 = H_{MNPQ}H^{MNPQ}$, and found a self-tuning solution. The action is

$$S = \int d^4x dy \left\{ \frac{1}{2} R_{(5)} - \frac{2 \cdot 4!}{H^2} - \Lambda_b - \Lambda_1 \delta(y) \right\} \tag{5}$$

where $R_{(5)}$ is the 5D Ricci scalar and we set the 5D fundamental mass $M = 1$. Λ_b and Λ_1 define two mass parameters: $k_b = \sqrt{-\Lambda_b/6}$, $k_1 = \Lambda_1/6$. The self-tuning solution with the Z_2 symmetry condition is given by

$$\beta(y) = \left[\frac{a}{k_b} \cosh(4k_b|y| + c) \right]^{-1/4} \tag{6}$$

where a is an integration constant from the field equation of A_{MNP} and c is another integration constant. Note that the brane tension Λ_1 is not fine-tuned. But as we will discuss, it must be bounded for the self-tuning solution to be possible. a is determined by the magnitude of the 4D Planck mass, and c is determined by the boundary condition at the brane $-\beta(y)' \big|_{y=0^+} = \Lambda_1/6$ where β is normalized as $\beta(0) = 1$. This self-tuning solution is connected to dS and AdS solutions [6]; hence it is a weak self-tuning solution. Since the solution is given in a closed form, it can help studying some properties of weak self-tuning solutions. The existence of the self-tuning solution in the RS-II setup can be recognized by looking at the equation

$$|\beta'| = \sqrt{\bar{k}^2 + k_b^2 \beta^2 - a^2 \beta^{10}}, \tag{7}$$

where $\bar{k}$ is the effective curvature of the 4D space, $\bar{k}^2 = +, 0, -$, corresponding to dS, flat and AdS, respectively. Note that the positive power of β in the a^2 term in Eq. (7) when one uses $1/H^2$. It would have a negative power β^{-6} if one used H^2 instead. The flat solution needs $\beta' \to 0$ as $\beta \to 0$. Therefore, the case H^2 does not satisfy this self-tuning solution criterion. On the other hand, $1/H^2$ term satisfies this condition. From this observation, one can find more self-tuning solutions [7]. Of particular interest among these is the self-tuning solution from the log function $\log(H^2)$. Since the solution with $1/H^2$ is given already, in the remainder of this talk we focus on this solution given in (6). The boundary condition at $y = 0$ determines c,

$$\tanh(c) = \frac{\Lambda_1}{\sqrt{-6\Lambda_b}}. \tag{8}$$

3. Blowing up solutions

The condition for the self-tuning solution, Eq. (8), is not always satisfied. It is because the region of tanh is limited to $[-1, +1]$. To see when it is not satisfied, let us note that the boundary condition at $y = 0^+$ requires

$$|\beta'|_{0^+} = \sqrt{\bar{k}^2 + k_b^2 - a^2} = \frac{\Lambda_1}{6} , \tag{9}$$

where $k_b^2 = \frac{-\Lambda_b}{6}$. Therefore, the condition that the flat solution is not allowed is

$$\frac{|\Lambda_1|}{6} > \sqrt{\frac{-\Lambda_b}{6}}, \tag{10}$$

or

$$\bar{k}^2 > a^2 \longrightarrow \overline{\Lambda} > 6a^2 M^2 \beta^{10}(0) \tag{11}$$

where $\overline{\Lambda}$ is the effective curvature of the 4D space. Thus, only the de Sitter space is allowed where the flat space solution is forbidden [9]. This situation is shown in Fig. 1 (b) as the shaded regions. The light(dark) shaded region is where Λ_1 is positive(negative). For comparison, in Fig. 1 (a), we show the 4D case where the flat space is possible only for $\Lambda_{\text{eff}} = 0$. This $\Lambda_{\text{eff}} = 0$ is blowed up to a finite region, $-\sqrt{-6\Lambda_b} < \Lambda_1 < +\sqrt{-6\Lambda_b}$, in Fig. 1 (b). Note that the AdS solution is also forbidden where the flat space solution is forbidden.

The region of the dS-only region is distinguished by positive and negative tensions. Because the negative tension leads to a phantom with negative kinetic energy [10], we adopt the parameters such that only a positive tension results.

This behavior of blowing up is a desirable property of weak self-tuning solutions. For the $1/H^2$, it was possible to show the blowing-up property as given above but it is difficult to see it if one does not have a closed form solution.
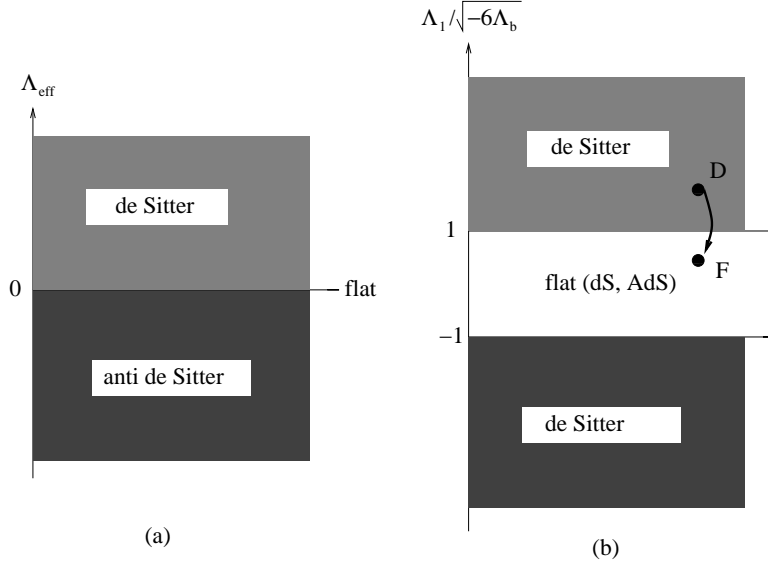
Fig. 1. A schematic illustration of the blow-up solution. The universe at the dS phase(Point D) goes into the flat-allowed region(Point F) by the parameter change at the brane.

4. Inflation with self-tuning solution

If only the de Sitter space is allowed for a finite range of parameter space in the Lagrangian, it is tempting to use it for a condition for inflation. By dynamics at the brane, we propose that after a sufficient inflation the parameter settles to the region where flat space solutions are possible. If this happens, say the universe goes from D to F in Fig. 1 (b), the universe at F starts from a de Sitter space solution. It seems to be possible because a sudden change of the Λ_1 parameter would not change the effective c.c. abruptly. But the F region also allows the flat space, and the key question is how the flat space is chosen from a dS solution after the sudden change of Λ_1. We hope to find a reasonable history for this scenario, which unifies the solutions of the large c.c.(for GUT phase inflation) and the vanishing c.c.(by the self-tuning solution). This is the dream we hoped to understand inflation on top of the vanishing cosmological constant from the time of the birth of inflation [11].

For this purpose, let us adopt the hybrid inflation at the brane. The key point of the hybrid inflation is to use multi fields among which there are an inflaton field ϕ and a waterfall field ψ [12]. This setup is particularly relevant for our scenario since at the brane Λ_1 can change instantaneously. The coupled potential of the fields ϕ, ψ living on the brane is taken as

$$V = V_0 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}(-m_\psi^2 + \lambda'\phi^2)\psi^2 + \frac{1}{2}\lambda'\psi^2\phi^2 + \Lambda_1 \tag{12}$$

where $V_0 = m_\psi^2\mu^2, m_\psi^2 = \lambda\mu^2$, and we treat μ^2 and m^2 the large and small parame-

ters, respectively. For ϕ greater than the critical value $\phi_c = \mu\sqrt{\lambda/\lambda'}$, the waterfall field is kept at origin $\psi = 0$, and there results a slow roll inflation. Requiring m^2 to be smaller than the Hubble parameter, $m^2 \ll H^2$, we obtain

$$\mu^2 \gg \sqrt{\frac{12}{\lambda}} \, m M_{Pl} \tag{13}$$

where $M_{Pl} = 1.2 \times 10^{19}$ GeV. The condition (13) also guarantees a sufficient inflation. The condition for forbidding a flat space solution is

$$\mu^4 > \sqrt{\frac{96}{\xi\lambda^2} M_{Pl}^2 \sqrt{ak_b}|\Lambda_b|} \tag{14}$$

where $\xi = O(1)$. For example, the eyeball numbers, $M = 8.4 \times 10^{16}$ GeV, $\mu > 2.2 \times 10^{15}$ GeV, $V_0 \sim \overline{\Lambda} > (1.86 \times 10^{13}$ GeV$)^4$, satisfy these conditions. Thus, we can obtain reasonable numbers from GUT related models. If ϕ crosses the critical value ϕ_c, the origin of ψ becomes the vacuum for tachyonic ψ, and the waterfall field ψ runs into the true vacuum immediately [12]. This parameter change occurs at the brane, and the brane tension after the waterfall field settles at the minimum is Λ_1. Namely, the universe settles at the point F, with a nonzero brane tension $\Lambda_1 > 0$, after the waterfall field finds the minimum. It has been shown that the parameters of the above hybrid inflation can fall in the region which GUT phase transitions allow [9].
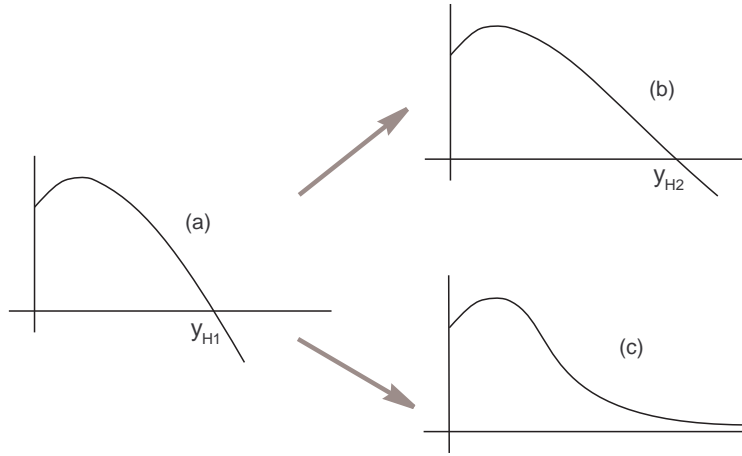

Fig. 2.   A schematic illustration of the transition from a dS space with horizon $H1$ (a) to another dS space with horizon $H2$ (b), or to a flat space (c).

Immediately after the hybrid inflation at the brane, our self-tuning solution is expected to be a dS one in the flat allowed blowing-up region. Then, one can imagine that the dS solution undergoes to a solution with a time-dependent c.c.

But the a closed form dS solution is not obtained. At the moment, the best we can obtain is the existence of the time-dependent c.c. For this, we studied a step function change of c.c. with a metric with time-dependent $b(t)$ [13]. As illustrated in Fig. 2, one anticipates the solution with the curvature change, i.e. the horizon distance changes from y_{H1} (a) to y_{H2} (b), or to ∞ (c). We found that in general there exist solutions for any y_{H2}. Therefore, classical physics does not determine the path. Here, Hawking's probabilistic interpretation [14] is applicable. In our setup, the initial condition after inflation is well defined, i.e. Point F in Fig. 1 (b). Then, it is shown that the probability to choose the flat space, i.e. Fig. 2 (c), is infinitely larger than choosing any other space [14].

5. Conclusion

In conclusion,

 (i) We tried to unify two vacuum energy problems, one the sufficient inflation and the other the vanishing cosmological constant.
(ii) A weak self-tuning solution was used for this scenario for the ultimately vanishing cosmological constant. It is possible for some range of inflaton field parameters for inflation to happen, $\Lambda_1 > \sqrt{-\Lambda_b/6}$.
(iii) For inflation at the brane and the exit from inflation, we showed the possibility in the hybrid inflation model.
(iv) The choice of the flat space after inflation is hoped to be solved classically. If classical physics does not determine the path, quantum corrections must choose the ultimate late universe.

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